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Duality for $SU \times SO$ and $SU \times Sp$ via branes

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Abstract: Using a six-orientifold, fourbranes and four fivebranes in type IIA string theory we construct $\mathcal{N}=1$ supersymmetric gauge theories in four dimensions with product group $SU(M) \times SO(N)$ or $SU(M) \times Sp(2N)$, a bifundamental flavor and quarks. We obtain the Seiberg dual for these theories and rederive it via branes. To obtain the complete dual group via branes we have to add semi-infinite fourbranes. We explain the addition of fourbranes as a reverse of higgsing.

Keywords: Duality in Gauge Field Theories, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Dirichlet branes have proved to be an extremely useful tool for obtaining non-perturbative information about gauge theories. A variety of brane constructions are available, which allow to induce a wide spectrum of gauge theories in the world-volume of the Dirichlet branes. We will be interested in configurations of Dirichlet fourbranes ending on Neveu-Schwarz fivebranes in Type IIA string theory. These configurations can be organized such that some supersymmetry is preserved [1]. When all the fivebranes are parallel, 1/4 of the initial supersymmetries survives and the theory describing the low energy effective action on the branes is an $\mathcal{N}$=2 four-dimensional gauge theory. One can reduce further to $\mathcal{N}$=1 by rotating the fivebranes in the orthogonal directions an $SU(2)$ angle [2]. Quantum corrections can be incorporated in this picture by lifting the brane configuration to M-theory [3]. Then the singular intersections of fivebranes and fourbranes are smoothed out and we obtain a single M-fivebrane wrapped around a Riemann surface with four uncompactified world-volume dimensions. For configurations preserving 1/4 of the supersymmetry this Riemann surface coincides with the Seiberg-Witten curve describing the Coulomb branch of $\mathcal{N}$=2 gauge theories [4]. Even in the Type IIA framework, the brane construction of gauge theories allows to derive very non-trivial information. The Seiberg dual of a given $\mathcal{N}$=1 gauge theory [5] can be obtained from certain brane moves [6].

We will derive the Seiberg dual for an $\mathcal{N}$=1 $SU(M) \times SO(N)$ or $SU(M) \times Sp(2N)$ gauge theory with a bifundamental flavor and quarks. This case has not yet been analyzed. We will obtain the dual theory first by field theory considerations and then by brane moves. This case offers a nice check for the brane approach to gauge theories. In order to obtain many of the known dual pairs, a superpotential must be added to the electric theory which truncates the set of chiral operators. On the other hand, the only possible obstruction to perform the mentioned brane moves is when we have to cross parallel branes. We find in our case a one to one correspondence between configurations with several parallel branes and situations in which the superpotential does not truncate the chiral ring. This will be the subject of sections 2 and 3.

In [7] the Seiberg dual for an $SU(N_1)\times SU(N_2)\times SU(N_3)$ was rederived via branes. They observed that branes predicted a smaller dual group. However the mismatch could be cured by adding a number of fourbranes to the dual configuration without modifying the linking numbers. We will encounter a similar problem. We will propose that the addition of fourbranes can be understood as a reverse of higgsing. In section 4 we will find a deformation of the electric theory which higgses the dual magnetic theory to the result derived from branes. We justify why such a deformation should be switched on by analyzing brane moves which correspond to dualize a single factor group. We treat in detail the case $SU(M)\times SO(N)$ and in section 5 extend briefly the results to $SU(M)\times Sp(2N)$.

2. Brane configuration

Our first ingredient is an orientifold sixplane extending in the $(x_0, x_1, x_2, x_3, x_7, x_8, x_9)$ directions. Thus the brane configurations which we will consider must be $\mathbb{Z}_2$ symmetric in the (x_4, x_5, x_6) directions. We will use four NS-fivebranes with world-volume along $(x_0, x_1, x_2, x_3, x_4, x_5)$ and fourbranes suspended between them expanding in (x_0, x_1, x_2, x_3) and with finite extent in the x_6 direction. In order to obtain gauge theories with $\mathcal{N}$=1 supersymmetry in the macroscopic dimensions of the fourbranes, we rotate the fivebranes an $SU(2)$ angle from (x_4, x_5) towards (x_8, x_9) [2]. The leftmost fivebrane A will be tilted at an angle θ_2, the interior fivebrane B at an angle θ_1. We place M fourbranes between the A and B fivebranes and N fourbranes between B and its mirror C. In addition there will be F sixbranes parallel to the A fivebrane and G sixbranes parallel to the B fivebrane in the (x_4, x_5, x_8, x_9) space and extending in $(x_0, x_1, x_2, x_3, x_7)$. The rest of the configuration is determined by the $\mathbb{Z}_2$ action of the orientifold (see fig. 1).

For an orientifold sixplane of positive Ramond charge, this configuration gives rise to an $\mathcal{N}$=1 gauge theory with product group $SU(M)\times SO(N)$ and the following matter content: fields X and $\widetilde{X}$ forming a flavor in the bifundamental representation, F flavors Q, $\widetilde{Q}$ transforming in the fundamental representation of $SU(M)$ and $2G$ chiral fields Q' in the vector representation of $SO(N)$ [8]. For an orientifold sixplane of negative Ramond charge we obtain a gauge theory with group $SU(M)\times Sp(2N)$ and the same matter content. We will concentrate on the product group $SU(M)\times SO(N)$. The analysis of $SU(M)\times Sp(2N)$ will be very similar and we will just make some remarks referring to it in section 5.

We want to determine the superpotential associated to our gauge theory. A conve-

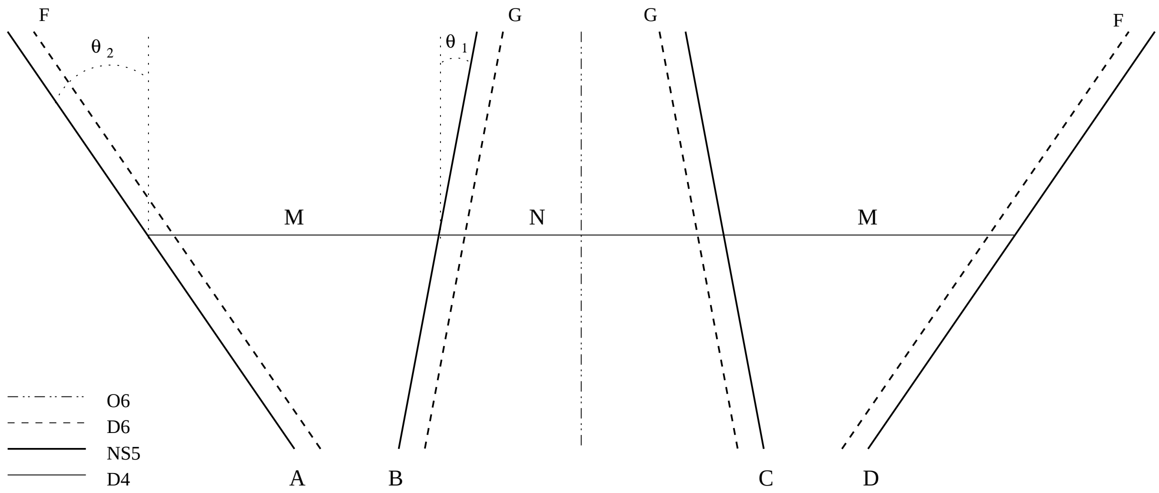

Figure 1: The brane configuration corresponding to an $\mathcal{N}$=1 $SU(M) \times SO(N)$ theory.

nient approach is to take as reference the superpotential for a brane configuration with additional massless tensor matter, add the mass term for the tensor field implied by the fivebrane rotation and integrate out this field. We will do the analysis separately for the tensor fields coming from the $SU(M)$ and $SO(N)$ sectors. The fourbranes suspended between the A and B fivebranes give rise to the $SU(M)$ factor group. When $\theta_1 = \theta_2$ there is an additional massless $SU(M)$ adjoint field, ϕ, whose expectation values move along the world-volume of the fivebranes. For arbitrary angles the superpotential for the SU sector will be $W = X\phi\widetilde{X} + \mu\text{Tr}\phi^2$, where the field ϕ gets a mass $\mu = \tan(\theta_2 - \theta_1)$ [2]. There is no coupling between ϕ and the F quarks since we are considering sixbranes parallel to the A fivebrane. Integrating out ϕ we get a superpotential

$$W_{SU} = \frac{-1}{4\tan(\theta_2 - \theta_1)} \left(\text{Tr}(X\widetilde{X})^2 - \frac{1}{M}(\text{Tr}X\widetilde{X})^2 \right) . \tag{2.1}$$

Fourbranes between the B and the C fivebrane give rise to the $SO(N)$ factor group. When $\theta_1 = 0$ we have an additional massless chiral field in the adjoint representation, ϕ_A. When $\theta_1 = \pi/2$ the B fivebrane and its dual are also parallel and we get additional massless matter, transforming this time in the symmetric representation of $SO(N)$, ϕ_S [9]. The bifundamental field couples to both ϕ_A and ϕ_S, which for arbitrary θ_1 are massive. The associated superpotential for the SO sector is

$$W = X\phi_A\widetilde{X} + X\phi_S\widetilde{X} + \mu'\text{Tr}\phi_A^2 - \frac{1}{\mu'}\text{Tr}\phi_S^2 \, , \tag{2.2}$$

where $\mu' = \tan\theta_1$.[1] Since the $2G$ $SO(N)$ chiral vector fields come from sixbranes parallel to the fivebranes, we will again suppose that there is no coupling between them and ϕ_A, ϕ_S. We will present a more careful discussion of this point in section 4. Integrating

[1] The mass for ϕ_S is $\tan(\pi/2 + \theta_1) = -1/\mu'$.

out both tensor fields we get

$$W_{SO} = \frac{-1}{4\tan 2\theta_1}\mathrm{Tr}(X\widetilde{X})^2 + \frac{1}{4\sin 2\theta_1}\mathrm{Tr}X\widetilde{X}\widetilde{X}X\,. \tag{2.3}$$

The final answer for the superpotential is then

$$W = W_{SU} + W_{SO} = a\mathrm{Tr}(X\widetilde{X})^2 + b\mathrm{Tr}X\widetilde{X}\widetilde{X}X + c(\mathrm{Tr}X\widetilde{X})^2, \tag{2.4}$$

where

$$a = -\frac{1}{4}\left(\frac{1}{\tan(\theta_2-\theta_1)} + \frac{1}{\tan 2\theta_1}\right), \qquad b = \frac{1}{4\sin 2\theta_1}\,, \qquad c = \frac{1}{4M\tan(\theta_2-\theta_1)}\,. \tag{2.5}$$

Using the F-term equations for the superpotential

$$\begin{aligned} a\widetilde{X}X\widetilde{X} + b\widetilde{X}\widetilde{X}X + c\widetilde{X}\ \mathrm{Tr}X\widetilde{X} &= 0\,, \\ aX\widetilde{X}X + bXX\widetilde{X} + cX\ \mathrm{Tr}X\widetilde{X} &= 0\,, \end{aligned} \tag{2.6}$$

we can deduce the chiral mesons of the theory. For generic values of the coefficients a and b we get: $M_0 = Q\widetilde{Q}$, $M_1 = Q\widetilde{X}X\widetilde{Q}$, $M_2 = Q\widetilde{X}X\widetilde{X}X\widetilde{Q}$, $M_0' = Q'Q'$, $M_1' = Q'X\widetilde{X}Q'$, $M_2' = Q'X\widetilde{X}\widetilde{X}XQ'$, $P_0 = Q\widetilde{X}Q'$, $P_1 = Q\widetilde{X}X\widetilde{X}Q'$, $\widetilde{P}_0 = \widetilde{Q}XQ'$, $\widetilde{P}_1 = \widetilde{Q}X\widetilde{X}XQ$, $R_1 = Q\widetilde{X}\widetilde{X}Q$, $\widetilde{R}_1 = \widetilde{Q}XX\widetilde{Q}$. For particular values of a and b the F-term equations can fail to truncate the chiral mesons to a finite set. This situation occurs when $a = 0$; then mesons containing $(\widetilde{X}X)^k$ are allowed for any k. Using (2.5) this corresponds to $\theta_1 = -\theta_2$, i.e. when the A and C fivebranes are parallel. In order to discard mesons containing the combination $\widetilde{X}X\widetilde{X}\widetilde{X}$ it was necessary to use

$$(a^2 - b^2)\widetilde{X}X\widetilde{X}\widetilde{X} = c(b-a)\widetilde{X}\widetilde{X}\ \mathrm{Tr}X\widetilde{X}, \tag{2.7}$$

which can be easily deduced from the F-term equations. Analogous relations hold for $\widetilde{X}\widetilde{X}X\widetilde{X}$, $X\widetilde{X}XX$ and $XX\widetilde{X}X$. These relations are trivial identities when $a = b$. When $a = -b$ they just constrain the product of some mesons with $\mathrm{Tr}X\widetilde{X}$ to be zero in the chiral ring. From (2.5), $a = b$ implies $\theta_2 = 0$ and $a = -b$ implies $\theta_2 = \pi/2$. These two situations correspond to the leftmost fivebrane A and its mirror D being parallel. When the A and B fivebranes are parallel, or B and C are parallel, we get additional tensor fields becoming massless and the set of chiral mesons also changes. Thus the stated set of chiral mesons is only valid when there are no parallel fivebranes.

In the next sections we will be interested in obtaining the Seiberg dual for our $\mathcal{N}$=1 $SU(M) \times SO(N)$ theory. We will derive it first from field theory methods and then from brane moves. The field theory derivation will be valid only when the set of chiral mesons is that stated above. On the other hand, the brane moves necessary to recover the dual theory will involve reversing the order of all fivebranes and sixbranes and this process is only well defined when there are no parallel fivebranes in our configuration. Notice that in our set-up parallel fivebranes imply sixbranes parallel to more than one fivebrane. Since the derived finite set of chiral mesons is valid if and only if there are no parallel fivebranes, we have an additional check for the validity of the brane derivation of Seiberg dualities.

The anomaly-free global symmetry group of our theory is

$$SU(F)_L \times SU(F)_R \times SU(2G) \times U(1)_R \times U(1)_B \times U(1)_X\,. \tag{2.8}$$

The brane diagram of fig. 1 does not exhibit the full $SU(2G)$ flavor symmetry. By bringing sixbranes over fivebranes we could obtain at most $SU(G) \times SU(G)$ [7]. However (2.8) is the global symmetry group for the superpotential (2.4) and thus it is the group we should consider in deriving the Seiberg dual theory. Another restriction of the brane construction is that we always get an even number of SO vectors, $2G$. The results of next section are valid for the general case where the SO factor has G' vectors just by substituting $2G$ by G'. The transformation properties of the matter fields under the gauge and global symmetry groups are summarized in table 1.

	SU(M)	SO(N)	SU(F)$_L$	SU(F)$_R$	SU($2G$)	U(1)$_R$	U(1)$_B$	U(1)$_X$
Q	M	1	F	1	1	$1-\frac{2M-N}{2F}$	$\frac{1}{M}$	0
$\widetilde{Q}$	$\overline{M}$	1	1	F	1	$1-\frac{2M-N}{2F}$	-$\frac{1}{M}$	0
Q'	1	N	1	1	$2G$	$1-\frac{N-M-2}{2G}$	0	0
X	M	N	1	1	1	$\frac{1}{2}$	$\frac{1}{M}$	1
$\widetilde{X}$	$\overline{M}$	N	1	1	1	$\frac{1}{2}$	-$\frac{1}{M}$	-1

Table 1: Matter content of the electric theory.

3. Dual theory

We propose that the dual theory has gauge group $SU(\widetilde{M}) \times SO(\widetilde{N})$ with $\widetilde{M} = 4F + 4G - M + 4$ and $\widetilde{N} = 8G + 4F - N + 8$. The matter content is given by fields Y and $\widetilde{Y}$ forming a flavor in the bifundamental representation, F flavors q, $\widetilde{q}$ transforming in the fundamental representation of $SU(\widetilde{M})$ and $2G$ fields q' in the vector representation of $SO(\widetilde{N})$. In addition there will be singlets M_i, M'_i with $i = 0,1,2$, $P_j, \widetilde{P}_j$ with $j = 0,1$ and $R_1, \widetilde{R}_1$ in one to one correspondence with the chiral mesons of the electric theory. The matter fields transform under the symmetries as indicated in table 2.

The dual superpotential is

$$\begin{aligned} W = {} & \mathrm{Tr}(Y\widetilde{Y})^2 + \mathrm{Tr}Y\widetilde{Y}\widetilde{Y}Y + (\mathrm{Tr}Y\widetilde{Y})^2 + M_0\widetilde{q}(Y\widetilde{Y})^2 q + M_1\widetilde{q}Y\widetilde{Y}q + \\ & + M_2\widetilde{q}q + M'_0 q'Y\widetilde{Y}\widetilde{Y}Yq' + M'_1 q'Y\widetilde{Y}q' + M'_2 q'q' + P_0 q\widetilde{Y}Y\widetilde{Y}q' + \\ & + P_1 q\widetilde{Y}q' + \widetilde{P_0}\widetilde{q}Y\widetilde{Y}Yq' + \widetilde{P_1}\widetilde{q}Yq' + R_1 q\widetilde{Y}\widetilde{Y}q + \widetilde{R_1}\widetilde{q}YY\widetilde{q}\,, \end{aligned} \tag{3.1}$$

where for simplicity we have ignored (dimensionful) coefficients in front of each term.

	$SU(\widetilde{M})$	$SO(\widetilde{N})$	$SU(F)_L$	$SU(F)_R$	$SU(2G)$	$U(1)_R$	$U(1)_B$	$U(1)_X$
q	$\widetilde{M}$	1	$\overline{F}$	1	1	$1-\frac{2\widetilde{M}-\widetilde{N}}{2F}$	$\frac{1}{\widetilde{M}}$	$\frac{\widetilde{M}+M-2F}{\widetilde{M}}$
$\widetilde{q}$	$\overline{\widetilde{M}}$	1	1	$\overline{F}$	1	$1-\frac{2\widetilde{M}-\widetilde{N}}{2F}$	$-\frac{1}{\widetilde{M}}$	$-\frac{\widetilde{M}+M-2F}{\widetilde{M}}$
q'	1	$\widetilde{N}$	1	1	$\overline{2G}$	$1-\frac{\widetilde{N}-\widetilde{M}-2}{2G}$	0	0
M_0	1	1	F	F	1	$-2+\frac{2\widetilde{M}-\widetilde{N}}{F}$	0	0
M_1	1	1	F	F	1	$-1+\frac{2\widetilde{M}-\widetilde{N}}{F}$	0	0
M_2	1	1	F	F	1	$\frac{2\widetilde{M}-\widetilde{N}}{F}$	0	0
M'_0	1	1	1	1	sym	$-2+\frac{\widetilde{N}-\widetilde{M}-2}{G}$	0	0
M'_1	1	1	1	1	sym $\oplus$ asym	$-1+\frac{\widetilde{N}-\widetilde{M}-2}{G}$	0	0
M'_2	1	1	1	1	sym	$\frac{\widetilde{N}-\widetilde{M}-2}{G}$	0	0
P_0	1	1	F	1	$2G$	$-\frac{3}{2}+\frac{2\widetilde{M}-\widetilde{N}}{2F}+\frac{\widetilde{N}-\widetilde{M}-2}{2G}$	0	-1
P_1	1	1	F	1	$2G$	$-\frac{1}{2}+\frac{2\widetilde{M}-\widetilde{N}}{2F}+\frac{\widetilde{N}-\widetilde{M}-2}{2G}$	0	-1
$\widetilde{P}_0$	1	1	1	F	$2G$	$-\frac{3}{2}+\frac{2\widetilde{M}-\widetilde{N}}{2F}+\frac{\widetilde{N}-\widetilde{M}-2}{2G}$	0	1
$\widetilde{P}_1$	1	1	1	F	$2G$	$-\frac{1}{2}+\frac{2\widetilde{M}-\widetilde{N}}{2F}+\frac{\widetilde{N}-\widetilde{M}-2}{2G}$	0	1
R_1	1	1	sym	1	1	$-1+\frac{2\widetilde{M}-\widetilde{N}}{F}$	0	-2
$\widetilde{R}_1$	1	1	1	sym	1	$-1+\frac{2\widetilde{M}-\widetilde{N}}{F}$	0	2
Y	$\widetilde{M}$	$\widetilde{N}$	1	1	1	$\frac{1}{2}$	$\frac{1}{\widetilde{M}}$	$\frac{M-2F}{\widetilde{M}}$
$\widetilde{Y}$	$\overline{\widetilde{M}}$	$\widetilde{N}$	1	1	1	$\frac{1}{2}$	$-\frac{1}{\widetilde{M}}$	$-\frac{M-2F}{\widetilde{M}}$

Table 2: Matter content of the magnetic theory.

The usual test for the duality ansatz are the t'Hooft anomaly matching conditions for the global symmetry group. The anomalies computed with the fermions of the electric theory must match those computed with the fermions of the magnetic theory.

Indeed we find for both theories

$$\begin{aligned}
\mathrm{U}(1)_R: &\quad -M^2+MN-\tfrac{N^2}{2}+\tfrac{3N}{2}-1 \\
\mathrm{U}(1)_R^3: &\quad -2MF(\tfrac{2M-N}{2F})^3-2GN(\tfrac{N-M-2}{2G})^3- \\
&\quad -\tfrac{1}{4}MN+M^2-1+\tfrac{N(N-1)}{2} \\
\mathrm{SU}(F)^3: &\quad Md_3(F) \\
\mathrm{SU}(F)^2\mathrm{U}(1)_R: &\quad -M\tfrac{2M-N}{2F}d_2(F) \\
\mathrm{SU}(F)^2\mathrm{U}(1)_B: &\quad d_2(F) \\
\mathrm{SU}(F)^2\mathrm{U}(1)_X: &\quad 0 \\
\mathrm{SU}(2G)^3: &\quad Nd_3(2G) \\
\mathrm{SU}(2G)^2\mathrm{U}(1)_R: &\quad -N\tfrac{N-M-2}{2G}d_2(2G) \\
\mathrm{U}(1)_B^2\mathrm{U}(1)_R: &\quad -2 \\
\mathrm{U}(1)_X^2\mathrm{U}(1)_R: &\quad -MN \\
\mathrm{U}(1)_R\mathrm{U}(1)_B\mathrm{U}(1)_X: &\quad -N
\end{aligned}$$

where $d_3(F)$ and $d_2(F)$ are the cubic and quadratic $SU(F)$ Casimirs of the fundamental representation. It is interesting to observe that the dual theory for $SU(M)\times SO(N)$ has much higher rank than the dual of an $SU(M)\times SU(N)$ theory with analogous matter content [10].

4. Brane moves

The electric brane configuration is shown in fig. 1. To find the dual theory, we reverse the order of the sixbranes as well as the fivebranes using the linking number conservation argument given in [1]. The orientifold plane is treated according to its Ramond charge, i.e. as a set of four sixbranes. With these rules we get the configuration shown in fig.2.

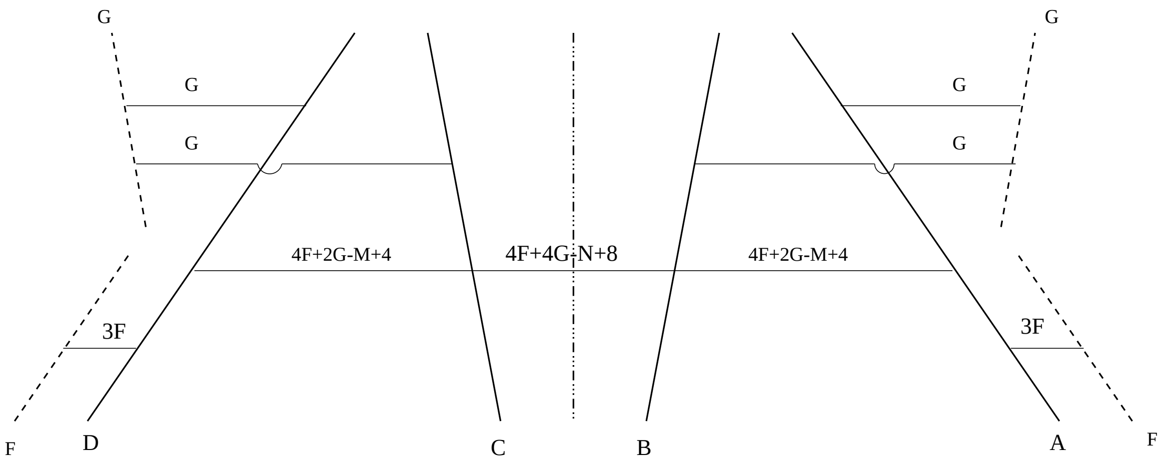

Figure 2: Brane configuration obtained by reversing the order of all branes.

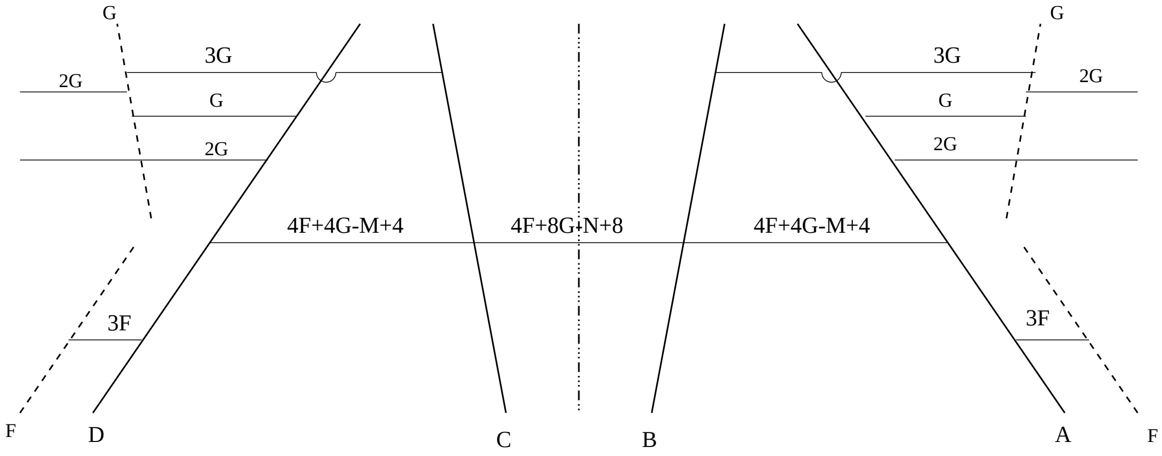

Figure 3: Dual brane configuration after adding $4G$ full fourbranes.

We obtain a gauge group $SU(\widetilde{M}') \times SO(\widetilde{N}')$ with $\widetilde{M}' = 4F + 2G - M + 4$ and $\widetilde{N}' = 4G+4F-N+8$, which does not coincide with that derived in the previous section. This mismatch can be cured by adding $4G$ full fourbranes to the dual configuration as shown in fig. 3. This does not affect the linking numbers. An analogous problem was encountered in [7] when rederiving the Seiberg dual for an $SU(N_1)\times SU(N_2)\times SU(N_3)$ gauge theory from brane moves. The brane configuration for that case is very similar to ours, it contains also four fivebranes. The number of fourbranes they had to add was twice the number of sixbranes placed between the second and the third fivebrane. We get the same result.

In the rest of this section we want to propose an explanation for the necessity of adding full fourbranes to recover the conjectured dual theory.[2] We will show that there exists a deformation of the electric theory that higgses the dual theory proposed in section 3 down to the result derived from brane moves. Deformations of the electric theory superpotential associated to mesons correspond generically to higgsing in the dual theory. Particular cases of meson deformations are those generated by M_0 and M_0', which give masses to the quarks $Q, \widetilde{Q}$ and Q' of the electric theory. They have a simple geometrical interpretation which corresponds to change the position of the sixbranes in the orthogonal directions to the fivebranes. However deformations of the superpotential generated by higher mesons do not have a clear geometrical interpretation. Thus from the brane configuration used to derive the electric theory, we can not determine a priori if some of these deformations are switched on.

In the dual brane configuration of fig. 2 we have $3F$ fourbranes suspended between the A fivebrane and its set of parallel sixbranes. The fourbranes can slide in the two directions shared by the fivebrane and the sixbranes. This can be understood as giving expectation values to the diagonal components of the M_i mesons, with $i = 0, 1, 2$ [6]. There are G fourbranes connecting the B fivebrane and its set of parallel sixbranes. This number is sufficient to provide the $2G$ fields q' transforming in the vector representation

[2] See also [11] for a reinterpretation of the additional fourbranes using a different approach

of $SO(\widetilde{N})$, but it seems that some of the M_i' mesons are missing. Based on this heuristic argument we will consider that the superpotential associated to the electric brane configuration of fig. 1 is actually $W + \Delta W$, with W given by (2.4) and ΔW a certain deformation generated by the mesons M_1' and M_2'. The superpotential of the dual theory will thus be

$$\begin{aligned} W = &\ \mathrm{Tr}(Y\widetilde{Y})^2 + \mathrm{Tr}Y\widetilde{Y}\widetilde{Y}Y + (\mathrm{Tr}Y\widetilde{Y})^2 + M_1'q'Y\widetilde{Y}q' + \\ &+ M_2'q'q' - m_1M_1' - m_2M_2' + \dots\,, \end{aligned} \tag{4.1}$$

where the dots stand for the other terms in (3.1).

The dual group $SU(\widetilde{M}) \times SO(\widetilde{N})$ differs from that obtained from brane moves by $\widetilde{M} = \widetilde{M}' + 2G$, $\widetilde{N} = \widetilde{N}' + 4G$. It will be sufficient to show that there exists a higgsing from $SU(\widetilde{M}) \times SO(\widetilde{N})$ to $SU(\widetilde{M}-1) \times SO(\widetilde{N}-2)$ without changing the matter content. Then by iterating this process $2G$ times we will arrive at the desired result. With this in mind we now study the superpotential (4.1) when $(m_1)_{\alpha\beta} = (m_2)_{\alpha\beta} = \delta_{\alpha 1}\delta_{\beta 1}$, where α and β are $SU(2G)$ flavor indices. The F-term equations for M_1' and M_2' are

$$q_\alpha' Y\widetilde{Y}q_\beta' = \delta_{\alpha 1}\delta_{\beta 1}\,, \qquad q_\alpha' q_\beta' = \delta_{\alpha 1}\delta_{\beta 1}\,. \tag{4.2}$$

Assuming that all the singlet fields have zero expectation value, the F-term equations, (2.6) and (4.2), and the D-term equations are solved by

$$\begin{aligned} \langle Y\rangle_i^a = \langle\widetilde{Y}\rangle_{ai} &= \frac{1}{\sqrt{3}}\,(\,\delta_{a1}\delta_{i1} + i\delta_{a1}\delta_{i2}\,)\,, \\ \langle q_1'\rangle_i &= \frac{1}{\sqrt{3}}\,(\,2\delta_{i1} - i\delta_{i2}\,)\,, \end{aligned} \tag{4.3}$$

where $a = 1,..,\widetilde{M}$ and $i = 1,..,\widetilde{N}$ are the SU and SO indices respectively. These expectation values break the gauge group to $SU(\widetilde{M}-1) \times SO(\widetilde{N}-2)$ as expected. It remains to analyze the matter content of the higgsed theory. We get a bifundamental flavor for the higgsed theory from the fields Y, $\widetilde{Y}$. The D-term equations allow also to recover from Y, $\widetilde{Y}$ an $SO(\widetilde{N}-2)$ vector and an $SU(\widetilde{M}-1)$ flavor in the fundamental representation. Substituting the expectation values (4.3) one can see that the superpotential gives mass to this additional SU flavor, but not to the SO vector. Thus the higgsed theory has the same content of charged matter as the initial one, i.e. a bifundamental flavor, F fundamental flavors of $SU(\widetilde{M}-1)$ and $2G$ vector fields of $SO(\widetilde{N}-2)$.

We have considered a deformation generated by both M_1' and M_2'. It is easy to see that setting $m_2 = 0$ does not alter the previous result, it only changes the particular expectation value of q_1'. Thus it is a deformation generated by M_1' that seems to be necessarily switched on in the brane construction. In order to obtain a better understanding of this point, let us perform a brane move associated to dualize the $SU(M)$ factor group considering $SO(N)$ as a flavor group.

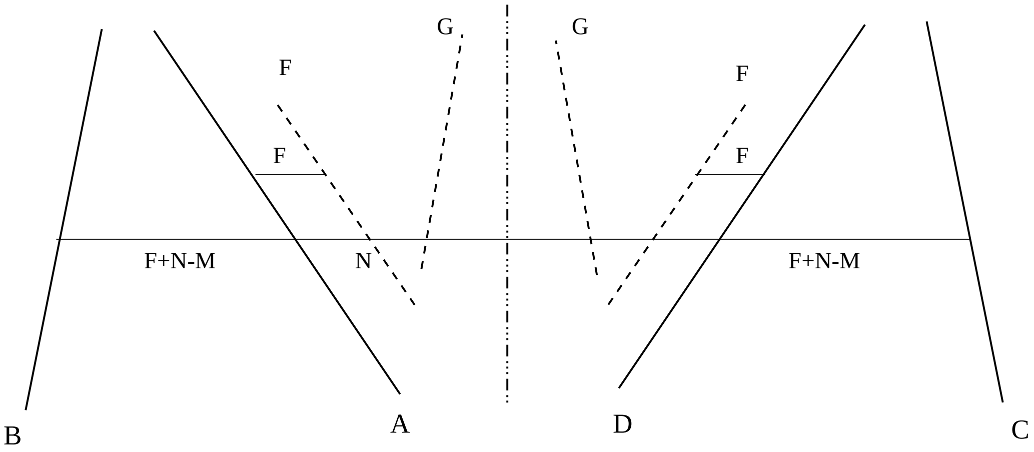

Figure 4: Brane configuration after interchanging A and B fivebranes.

This is done by moving the sixbranes parallel to the A fivebrane over B and then exchanging A and B as depicted in fig. 4. We have not moved the two groups of sixbranes in the central part of the diagram, which give rise to $2G$ $SO(N)$ vector fields, Q'. We observe that now the $2G$ central sixbranes are not parallel to the fivebranes that define the $SO(N)$ group, i.e. A and D. We perform now a further move corresponding to dualize the $SO(N)$ factor group keeping SU as a spectator. This is done by bringing the sixbranes to the left of the orientifold in fig. 4 over the D fivebrane, the sixbranes to the right of the orientifold over the A fivebrane and then exchanging A and D. The fourbranes created in this process between the two sets of G sixbranes and the A and D fivebranes can not slide along the world-volume of the fivebrane, since the mentioned fivebranes and sixbranes are not parallel. In terms of the dual SO theory this means that the singlet field associated with the $SO(N)$ meson $Q'Q'$ must be massive. A mass term for this singlet can only be obtained when the superpotential of the electric $SO(N)$ theory contains a quartic term in the fields Q'. An interesting remark is that fig. 4 realizes only an $SU(G)$ subgroup of the $SU(2G)$ flavor symmetry for the Q', and not $SU(G) \times SU(G)$ as fig. 1 suggests.

Let us compare the previous situation with that obtained by dualizing instead the $SO(N)$ group in the configuration of fig. 1. Then the fourbranes created between the two sets of G sixbranes and the B and C fivebranes can slide along the world-volume of the fivebrane, since the central sixbranes have been assumed to be parallel to B and C (see fig. 5). In the dual SO theory there must be a massless singlet. Therefore a quartic term in Q' will not be present in the electric theory.[3] However, according to the previous paragraph, the superpotential for the configuration in fig. 1 should be such that when we dualize the $SU(M)$ factor group the resulting dual superpotential does indeed contain a quartic term in the fields Q'.

[3]There is a subtlety here. The mentioned singlet transforms in the symmetric representation of $SU(2G)$, but only an $SU(G)$ subgroup of $SU(2G)$ is realized in the brane diagram. We can not rule out the presence of a restricted quartic term in Q' which would lift some components of the dual singlet reducing it to an $SU(G)$ adjoint.

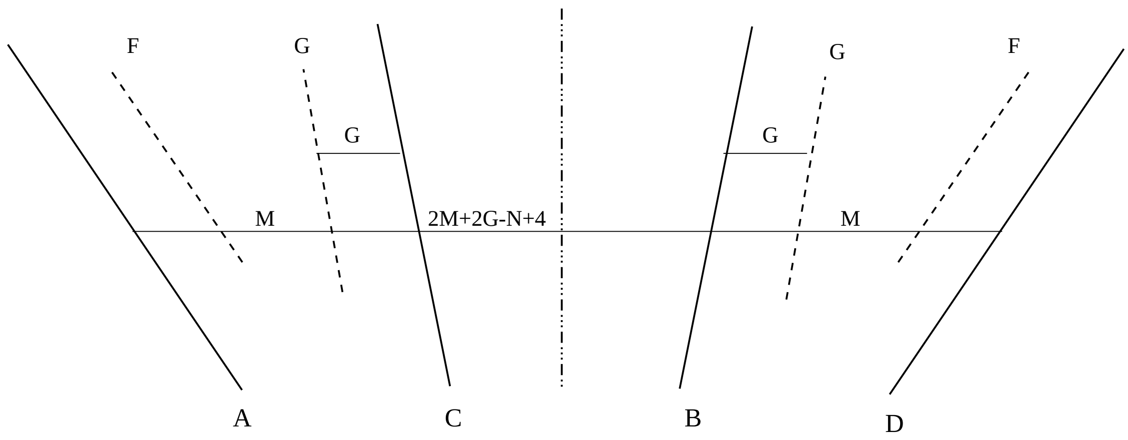

Figure 5: Brane configuration after interchanging B and C fivebranes.

This is precisely achieved by adding to the superpotential (2.4) a deformation generated by the meson M_1'

$$W = (X\widetilde{X})^2 + m_{\alpha\beta}\, Q'^{\alpha} X\widetilde{X} Q'^{\beta}, \tag{4.4}$$

where we have denoted all the terms appearing in (2.4) as $(X\widetilde{X})^2$. Dualizing the $SU(M)$ factor group [5] we get an $SU(F+N-M)\times SO(N)$ gauge theory with a bifundamental flavor, F SU quark flavors and $2G+2F$ SO vector fields. The additional $2F$ SO vectors have their origin in the $SU(M)$ chiral mesons $\widetilde{Q}X$ and $Q\widetilde{X}$. We will denote them by q_1' and q_2' respectively. The $SO(N)$ fields q_1' and q_2' do have a representation in the brane diagram of fig. 4. Since the set of F sixbranes is parallel to the A fivebrane, the fourbranes suspended between them can be moved arbitrarily far away from the intersection between the F sixbranes and the N central fourbranes. Thus strings joining the F sixbranes and the N central fourbranes will give rise to the $2F$ additional SO vector fields. This conclusion is of course not valid when the F sixbranes and the A fivebrane are not parallel [1]. The dual $SU(F+N-M)\times SO(N)$ theory contains also a singlet field M_Q and a field M_X transforming as the direct sum of the adjoint and the symmetric representations of $SO(N)$. These fields are in correspondence with the $SU(M)$ chiral mesons $Q\widetilde{Q}$ and $X\widetilde{X}$. The quartic superpotential for the bifundamental fields in (4.4) translates into a mass term for M_X in the dual theory

$$W = (M_X)^2 + m\, Q' M_X Q' + M_Q q\tilde{q} + q_1' Y \tilde{q} + q_2'\, \widetilde{Y} q + M_X Y\widetilde{Y}. \tag{4.5}$$

Integrating out this field, we get

$$W = (Y\widetilde{Y} + m\, Q'Q')^2 + M_Q q\tilde{q} + q_1' Y\tilde{q} + q_2'\widetilde{Y} q\,, \tag{4.6}$$

which contains a quartic term for the fields Q'.

We can obtain more information about the matrix $m_{\alpha\beta}$ by considering again the situation of fig. 5, which corresponds to dualize the $SO(N)$ factor group. The resulting dual group is $SU(M)\times SO(2M+2G-N+4)$ [5] [12]. The brane diagram implies that there are $F+G$ $SU(M)$ quarks. The additional SU quarks can only have their origin in the $SO(N)$ chiral mesons $Q'X$, $Q'\widetilde{X}$. We denote them by q and $\tilde{q}$ respectively; they form $2G$ $SU(M)$ flavors. The term $mQ'X\widetilde{X}Q'$ in (4.4) translates in the dual $SU(M)\times SO(2M+2G-N+4)$ theory into a mass term for q and $\tilde{q}$

$$m_{\alpha\beta}q^{\alpha}\tilde{q}^{\beta}\,. \tag{4.7}$$

Thus m has to be such that it gives mass to G of the new flavors. A way of choosing m with the above property and preserving an $SU(G)$ subgroup of the $SU(2G)$ flavor symmetry is

$$m_{\alpha\beta}\sim\begin{cases}\delta_{\alpha+G,\beta} & \alpha=1,..,G\,,\\ 0 & \text{otherwise}\,.\end{cases} \tag{4.8}$$

Notice that this expression for m makes sense because the meson M_1' transforms as the direct sum of the symmetric and antisymmetric representations of $SU(2G)$. A check for the proposed structure of m is the following. When substituted in (4.6) and after dualizing further the $SO(N)$ group, it must induce mass terms for all the components of the singlet associated with the $SO(N)$ meson $Q'Q'$. Remembering that the term in parenthesis in (4.6) was a short way of denoting several terms as in (2.4), we obtain

$$\begin{aligned} m_{\alpha\beta}Q'^{\alpha}_{i}Q'^{\beta}_{j}m_{\delta\gamma}Q'^{\delta}_{j}Q'^{\gamma}_{i} &\sim M^{\hat{\alpha},\hat{\delta}+G}M^{\hat{\delta},\hat{\alpha}+G},\\ m_{\alpha\beta}Q'^{\alpha}_{i}Q'^{\beta}_{j}m_{\gamma\delta}Q'^{\delta}_{j}Q'^{\gamma}_{i} &\sim M^{\hat{\alpha},\hat{\delta}}M^{\hat{\alpha}+G,\hat{\delta}+G}. \end{aligned} \tag{4.9}$$

with $\hat{\alpha},\hat{\delta}=1,..,G$, which indeed gives mass to all components of M.

The previous arguments can be applied to more generic brane configurations than those considered here. They suggest that, for configurations with several fivebranes and sixbranes, the superpotential includes generically quartic couplings between quarks and bifundamental fields. The presence of such terms does not sound surprising for brane configurations with sixbranes not parallel to the adjacent fivebranes. Our main result is that for configurations in which the sixbranes are parallel to one of the adjacent fivebranes, the superpotential also contains a term $Q\widetilde{X}X\widetilde{Q}$ coupling the quarks coming from the sixbranes and the bifundamental field coming from the parallel fivebrane. In particular for configurations with more than three fivebranes, such terms are unavoidable. These terms translate in the dual theory into terms linear in the singlet fields, which have the effect of higgsing. Besides the case treated in this paper, this explains why the brane approach to $SU(N_1)\times SU(N_2)\times SU(N_3)$ predicts a dual group of smaller rank than that derived by field theory arguments [7].

We would like to end this section with one additional comment. We have argued that the brane construction of our $SU(M)\times SO(N)$ theory corresponds to the modified superpotential (4.4) instead of (2.4). However the brane moves necessary to derive the dual theory have a field theory interpretation independent of what the concrete superpotential is. They can be seen as successive, seperate dualizations of the SU and

JHEP11(1998)020

SO gauge groups. In particular the brane moves that bring us from fig. 1 to fig. 2 correspond to dualize first SU, then SO, then again SU and finally again SO (or alternatively first SO, then SU and then again SO and SU). We can apply this chain of dualities to the $SU(M) \times SO(N)$ theory with the undeformed superpotential (2.4). After each step fields transforming in tensor representations appear. They are massive due to the quartic term in the bifundamental fields in the superpotential and can be integrated out. Thus we only need to use the known dualities for SU and SO groups with matter in the fundamental and vector representation respectively [5], [12]. We have checked that the dual theory derived in this way coincides with the one proposed in section 3, which is a very strong test for our conjectured dual theory.[4] These calculations are straightforward but rather lengthy and we will not include them here. However the first step, corresponding to dualize the $SU(M)$ group, has been explicitly analyzed above with the modified superpotential.

5. $SU(M) \times Sp(2N)$

We state briefly some results for the brane set-up in fig. 1 with an orientifold sixplane of negative Ramond charge. In this case we obtain an $\mathcal{N}=1$ theory with gauge group $SU(M) \times Sp(2N)$ and the same matter content as before. The superpotential derived from the brane configuration is

$$W = a\mathrm{Tr}(X\widetilde{X})^2 + b\mathrm{Tr}X\widetilde{X}\widetilde{X}X + c(\mathrm{Tr}X\widetilde{X})^2, \tag{5.1}$$

where

$$a = -\frac{1}{4}\left(\frac{1}{\tan(\theta_2-\theta_1)} + \frac{1}{\tan 2\theta_1}\right), \qquad b = \frac{-1}{4\sin 2\theta_1}, \qquad c = \frac{1}{4M\tan(\theta_2-\theta_1)}. \tag{5.2}$$

The mesons are the ones given in section 2 and also the global symmetry group. The transformation properties of the matter fields under the gauge and global symmetry groups are listed in table 3.

	SU(M)	Sp($2N$)	SU(F)$_L$	SU(F)$_R$	SU($2G$)	U(1)$_R$	U(1)$_B$	U(1)$_X$
Q	M	1	F	1	1	$1-\frac{M-N}{F}$	$\frac{1}{M}$	0
$\widetilde{Q}$	$\overline{M}$	1	1	F	1	$1-\frac{M-N}{F}$	-$\frac{1}{M}$	0
Q'	1	$2N$	1	1	$2G$	$1-\frac{2N-M+2}{2G}$	0	0
X	M	$2N$	1	1	1	$\frac{1}{2}$	$\frac{1}{M}$	1
$\widetilde{X}$	$\overline{M}$	$2N$	1	1	1	$\frac{1}{2}$	-$\frac{1}{M}$	-1

Table 3: Matter content of the electric theory.

[4]We thank the referee for suggesting this test to us.

JHEP11(1998)020

	$\mathrm{SU}(\widetilde{M})$	$\mathrm{Sp}(2\widetilde{N})$	$\mathrm{SU}(F)_L$	$\mathrm{SU}(F)_R$	$\mathrm{SU}(2G)$	$\mathrm{U}(1)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_X$
q	$\widetilde{M}$	1	$\overline{F}$	1	1	$1-\frac{\widetilde{M}-\widetilde{N}}{F}$	$\frac{1}{\widetilde{M}}$	$\frac{\widetilde{M}+M-2F}{\widetilde{M}}$
$\widetilde{q}$	$\overline{\widetilde{M}}$	1	1	$\overline{F}$	1	$1-\frac{\widetilde{M}-\widetilde{N}}{F}$	$-\frac{1}{\widetilde{M}}$	$-\frac{\widetilde{M}+M-2F}{\widetilde{M}}$
q'	1	$2\widetilde{N}$	1	1	$\overline{2G}$	$1-\frac{2\widetilde{N}-\widetilde{M}+2}{2G}$	0	0
Y	$\widetilde{M}$	$2\widetilde{N}$	1	1	1	$\frac{1}{2}$	$\frac{1}{\widetilde{M}}$	$\frac{M-2F}{\widetilde{M}}$
$\widetilde{Y}$	$\overline{\widetilde{M}}$	$2\widetilde{N}$	1	1	1	$\frac{1}{2}$	$-\frac{1}{\widetilde{M}}$	$-\frac{M-2F}{\widetilde{M}}$

Table 4: Matter content of the magnetic theory.

The dual theory has gauge group $SU(\widetilde{M}) \times Sp(2\widetilde{N})$ with $\widetilde{M} = 4F+4G-M-4$ and $\widetilde{N} = 2F+4G-N-4$. The field content of the dual theory and the transformation under the symmetries are indicated in table 4. Note that the mesons M_0' and M_2' are now in the antisymmetric representation of $SU(2G)$. The dual theory has a superpotential as in (3.1). As in the previous case the dual theory can be obtained from the known dualities for SU and Sp groups [5], [13] by dualizing first the SU factor, then the Sp factor and then SU and Sp again[5] (or alternatively first Sp, then SU and then again Sp and SU).

When we try to recover the dual theory from brane moves we get a smaller dual group $SU(\widetilde{M}') \times Sp(2\widetilde{N}')$ with $\widetilde{M}' = \widetilde{M}-2G$, $\widetilde{N}' = \widetilde{N}-2G$. We can cure this mismatch by adding $4G$ full fourbranes to the dual configuration. All the arguments presented in the previous section to explain this problem extend to the $SU \times Sp$ case. We can understand the addition of the fourbranes as a reverse of higgsing in the dual theory, induced by adding to the superpotential (5.1) of the electric theory a deformation generated by the meson M_1'.

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JHEP11(1998)020